Radius of the Incircle of a Triangle

Brian Rogers

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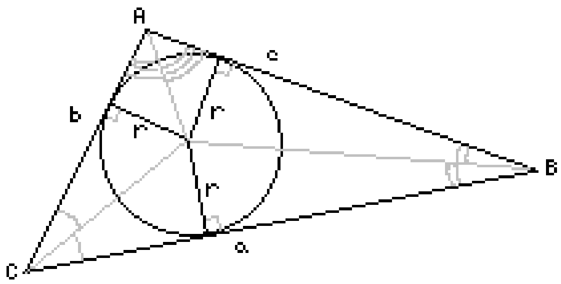

The center of the incircle of a triangle is located at the intersection of the angle bisectors of the triangle. Given the side lengths of the triangle, it is possible to determine the radius of the circle.

First, form three smaller triangles within the triangle, one vertex as the center of the incircle and the others coinciding with the vertices of the large triangle. Then, drop an altitude from the vertex at the incircle center for each smaller triangle. Each altitude segment, r, is a radius of the incircle. Use the fact that the sum of the areas of the smaller triangles is equal to the area of the larger triangle to obtain an expression for the radius.

$$\frac{1}{2}r \cdot a + \frac{1}{2}r \cdot b + \frac{1}{2}r \cdot c = A \tag{1}$$

$$\frac{1}{2}r\,(a+b+c) = A \tag{2}$$

$$r = \frac{2A}{a+b+c} \tag{3}$$

The area of the triangle A can be determined by Heron's Area Formula, given the semiperimeter $s = \dfrac{a+b+c}{2}$:

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)} \tag{4}$$