Fractions

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Fractions are used all the time in daily life. Half a sandwich $\left(\frac{1}{2}\right)$, two-thirds of a cup of water $\left(\frac{2}{3}\right)$, the third quarter of the football game $\left(\frac{3}{4}\right)$ and so on. A fraction is essentially a division problem. For instance, how can I divide *ONE* pie into *SIX* pieces? It's simple, each person gets $\frac{1}{6}$ of the pie. How can I split 5 dollars between four friends? $\frac{5}{4}$, or $1\frac{1}{4}$ dollars each!

1   Types of fractions

The most common types of fractions are simply known as **common fractions**. A common fraction has two parts: the top, or numerator, and the bottom, or denominator.

Oftentimes, a fraction represents a number less than one. For example, $\frac{5}{7}$, $\frac{10}{11}$, or $\frac{100}{999}$. A fraction that represents a number less than one is called a **proper fraction**. Remember, if the numerator (top) is less than the denominator (bottom), the fraction's value is less than one and it's a proper fraction.

Occasionally, you'll see a fraction where the numerator is greater than the denominator. This is called an **improper fraction**. Examples: $\frac{5}{3}$, $\frac{7}{4}$, $\frac{171}{113}$.

There is also another type of fractional number called a **mixed fraction** or **mixed number**. A mixed number has two parts: a whole number and a common fraction. For example, $5\frac{1}{7}$ ("five and one-seventh") or $2\frac{2}{9}$ ("two and two-ninths").

Any mixed number can be written as an improper fraction. The reverse is also true, any improper fraction can be written as a mixed number.

2   How to convert an improper fraction to a mixed number

1. Set up a division problem (num = numerator, denom = denominator):

   $denom\overline{)num}$

2. The whole number is the whole number answer to the division problem. The remainder is the numerator of the new fraction. (The new fraction has the same denominator as before.)

Example:
$\frac{17}{12}$

$$
\begin{array}{r}
1 \\
12\overline{)17} \\
-12 \\
\hline
5
\end{array}
$$

The mixed number will then be $1\frac{5}{12}$. 1 was the whole number, 5 was the remainder, and 12 was the denominator (same as before).

3 How to add (or subtract) two fractions

The rules say that you can only add fractions that have the same denominator. This is why we always "find the common denominator" between two fractions before adding/subtracting.

Let's say we want to add $\frac{7}{8}$ and $\frac{5}{12}$. The denominators are 8 and 12. We need to think of a number that is a multiple of both 8 and 12. 24 works, so let's call that our common denominator.

Now we want to make both denominators "look like" our common denominator. So how do we turn 8 into 24? Multiply by 3. But we can't just multiply the bottom by 3. The rules say that anything we multiply on the bottom, we have to multiply on the top as well. (Otherwise, we'd change the number.) So let's do it:

$$\frac{7}{8} \Rightarrow \frac{7 \times 3}{8 \times 3} \Rightarrow \frac{21}{24}$$

How do we make 12 look like 24? Multiply by 2.

$$\frac{5}{12} \Rightarrow \frac{5 \times 2}{12 \times 2} \Rightarrow \frac{10}{24}$$

Now we can add the numerators together (and keep the denominators the same):

$$\frac{21}{24} + \frac{10}{24} = \frac{21 + 10}{24} \Rightarrow \frac{31}{24}$$

The answer is $\frac{31}{24}$. This answer is perfectly acceptable, but you may want to write a mixed number instead. In that case, it would be $1\frac{7}{24}$.

Subtraction of fractions is exactly the same, except you subtract on the last step:

$$\frac{5}{7} - \frac{1}{3} = ???$$

The common denominator is 21 (a multiple of 3 and 7).

$$\frac{5 \times 3}{7 \times 3} - \frac{1 \times 7}{3 \times 7} \Rightarrow \frac{15}{21} - \frac{7}{21}$$

$$\frac{15 - 7}{21} = \frac{8}{21}$$

The answer is $\frac{8}{21}$.

4 How to multiply two fractions

Multiplying is usually considered "easier" than addition or subtraction. To multiply two fractions, just multiply straight across.

$$\frac{2}{3} \times \frac{7}{6} = \frac{2 \times 7}{3 \times 6} = \frac{14}{18}$$

Of course, there is one more thing. This fraction is not in "lowest terms." To get a fraction in lowest terms, we can reduce it by dividing the top and bottom by the same number. First, we have to find a number that goes into the numerator and denominator evenly. 2 goes into both 14 and 18 evenly, so let's divide:

$$\frac{14 \div 2}{18 \div 2} = \frac{7}{9}$$

The only number that goes into 7 and 9 evenly is 1, so we're done! The fraction is completely reduced, or in lowest terms.

5 How to divide two fractions

Division is almost the same as multiplication – with one twist! To divide two fractions, we flip the right side and multiply. "Flip" means switch the numerator and denominator ($\frac{2}{3}$ would become $\frac{3}{2}$).

Example:

$$\frac{1}{3} \div \frac{5}{6} \Rightarrow \frac{1}{3} \times \frac{6}{5}$$

$$\frac{1}{3} \times \frac{6}{5} = \frac{6}{15} \Rightarrow \frac{2}{5}$$

It's as simple as that.

6 Math with mixed numbers

What if you have to add, subtract, multiply, or divide using mixed numbers? The answer is, don't use mixed numbers... first convert to improper fractions and then proceed with your calculation.

Example:

$$2\frac{3}{5} + 1\frac{1}{10} = ???$$

Convert both mixed numbers into improper fractions.

$$2\frac{3}{5} = \frac{2 \times 5 + 3}{5} = \frac{13}{5}$$

$$1\frac{1}{10} = \frac{1 \times 10 + 1}{10} = \frac{11}{10}$$

Now we have a standard addition problem with fractions.

$$\frac{13}{5} + \frac{11}{10} = ???$$

Find the common denominator and add as usual.

$$\frac{13 \times 2}{5 \times 2} + \frac{11}{10} = \frac{26}{10} + \frac{11}{10} = \frac{26 + 11}{10} = \frac{37}{10}$$

The answer is $\frac{37}{10}$, or $3\frac{7}{10}$ as a mixed number.